SPRING MANUFACTURERS' RESEARCH ASSOCIATION

The Design of Garter Springs

Report No. 128

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A garter spring is a helical spring the ends of which are joined in such a way that the free spring is a ring with outside diameter D_{ro}, mean diameter D_r, and inside diameter D_{ri} (Fig. 1). It may operate in extension or compression.

1. The garter spring in extension

If the spring is fitted over a shaft with diameter $D_s > D_{ri}$, D_{ri} is increased to D_s by $\Delta D = D_s - D_{ri}$ and D_r by the same amount to $D_r + \Delta D$ so that the mean circumference of the ring is lengthened by $\Delta\pi D$. The spring load due to this extension is given by

$$P_1 = \pi\Delta DS$$

where S denotes the spring rate. If the spring is wound with initial tension, the corresponding load P_o has to be added to P_1 so that

$$P = P_o + P_1 = P_o + \pi\Delta DS. \qquad (1)$$

The extended spring is exerting a uniform radial pressure on the shaft.

If a cylinder with diameter D_r is subject to an internal pressure p_r in pounds per unit of length of its circumference it can be shown that the resultant load of the radial pressure p_r acting on half the cylinder is the same as if p_r acted on its diameter. This resultant load $p_r \times D_r$ produces a tangential load P in the cylinder so that (see Fig. 2)

$$2P = p_r D_r$$

$$\text{or} \quad p_r = \frac{2P}{D_r} \text{ lb/in.} \qquad (2)$$

If the circular axis of the garter spring with diameter $D_r + \Delta D$ is considered as a cylinder, the spring load P produces the radial pressure p_r on this cylinder. This pressure is given by (2) after replacing D_r by $D_r + \Delta D$.

By substituting P from (1), equation (2) becomes

$$p_r = \frac{2P_o}{D_r+\Delta D} + \frac{2\pi\Delta DS}{D_r+\Delta D} \cdot \qquad (3)$$

As the circumference πD_s of the shaft is smaller than the mean circumference $\pi(D_r+\Delta D)$ of the garter spring, the pressure p exerted on the shaft is larger than p_r. Obviously, the magnitudes of pressure are inversely proportional to the corresponding magnitude of circumference. Then,

$$\frac{p}{p_r} = \frac{\pi(D_r+\Delta D)}{\pi D_s}$$

or

$$p = \frac{D_r+\Delta D}{D_s} p_r.$$

Consequently, pressure p is obtained by multiplying (3) by $(D_r + \Delta D)/D_s$. The result is

$$p = 2\left[\frac{P_o}{D_s} + \frac{\pi\Delta D}{D_s} S\right]$$

or, as $\Delta D = D_s - D_{ri}$,

$$p = 2\left[\frac{P_o}{D_s} + \pi\left(1-\frac{D_{ri}}{D_s}\right) S\right]. \qquad (4)$$

2. The garter spring in compression

If a garter spring is fitted into a tube with inside diameter $D_t < D_{ro}$, the mean diameter of the spring is reduced by $\Delta D = D_{ro} - D_t$. As the spring is stressed in compression, there is no initial load P_o. In this case, the radial pressure exerted on the inside of the tube is given by

$$p = 2\pi\left(\frac{D_{ro}}{D_t} - 1\right) S. \qquad (5)$$

The Stress in Garter Springs

If a helical spring with mean diameter D, wire diameter d, spring index c = D/d, curvature correction factor K = (c + 0.2)/(c-1) and number of working coils n undergoes a change of length δ, the corresponding corrected stress is given by

$$q_{c1} = \frac{GK\delta}{n\pi cD} \cdot$$

In the case of the garter spring, where $\delta = \pi\Delta D$, this equation becomes

$$q_{c1} = \frac{\Delta D}{D} \frac{GK}{nc} .$$

With the ends joined, the originally straight axis of the spring is a circle, and the spring is subject to a bending moment

$$M = EI \frac{1}{R}$$

$$= EI \frac{2}{D_r \pm \Delta D} \qquad (6)$$

where the + or − sign pertains to the spring in tension or compression, respectively. The bending stiffness of a helical spring with length L and spring rate S is given by

$$EI = \frac{1}{2} \frac{L D^2 S}{1+2G/E}$$

or, as in this case $L = \pi(D_r \pm \Delta D)$, by

$$EI = \frac{\pi}{2} \frac{(D_r \pm \Delta D)}{1+2G/E} D^2 S.$$

Substituting this term and $S = dG/8nc^3$ in (6) gives

$$M = \frac{\pi D^2}{1+2G/E} \frac{dG}{8nc^3} .$$

This moment produces the torsional shear stress

$$q_{c2} = \frac{16 M K}{\pi d^3}$$

$$= \frac{16 K}{\pi d^3} \cdot \frac{\pi D^2}{1+2G/E} \cdot \frac{dG}{8nc^3}$$

$$= \frac{2}{1+2G/E} \cdot \frac{GK}{nc}$$

which is independent of the mean diameter $D_r \pm D$ of the garter spring in position as long as contact between the coils does not take place. (Fig. 1). For conventional spring steels with $G = 11.5 \times 10^6$ lb/sq.in. and $E = 30 \times 10^6$ lb/sq.in., the term $2/(1+2G/E)$ equals 1.13.

The total shear stress q_c is the sum of q_{c1} and q_{c2}, i.e.

$$q_c = \left[\frac{\Delta D}{D} + \frac{2}{1+2G/E}\right] \frac{GK}{nc} \qquad (7)$$

where $\Delta D = D_s - D_{ri}$ or $D_{ro} - D_t$ according to whether the spring is stressed in tension or compression, respectively.

If, in the first case, the spring is coiled with initial tension the stress

$$q_{ci} = \frac{8aP_o K}{\pi d^2}$$

has to be added to q_c.

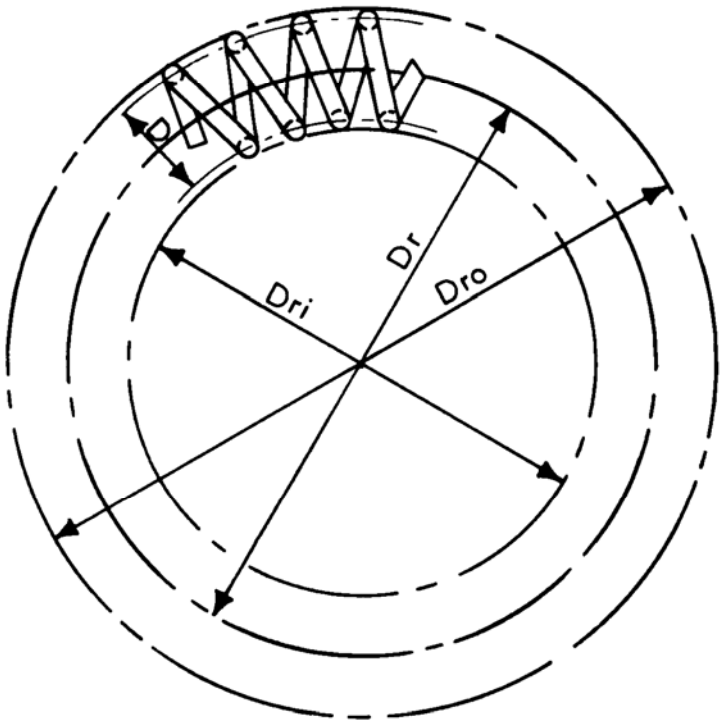

FIG.1.

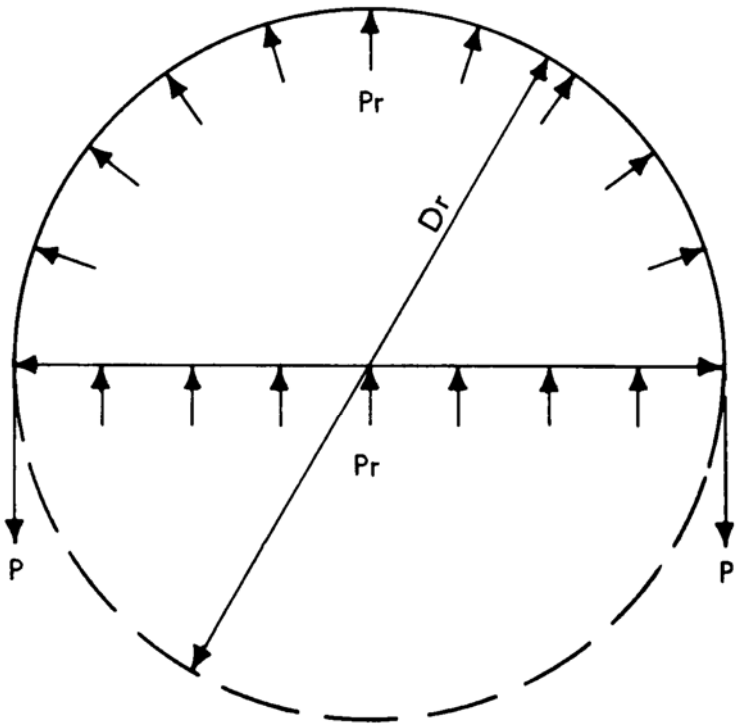

FIG.2.