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SPRING DESIGN DATA SHEET 6

THE DESIGN OF TORSION BARS

THE INSTITUTE OF SPRING TECHNOLOGY LTD

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Notation

a = reduction factor (see Figure 5)
d = diameter of cylindrical part of bar
d_e = overall diameter at shouldered end
d_h = inside diameter of hollow bar
d_l = root diameter of serrations at shouldered end
d_o = outside diameter of hollow bar
E = modulus of elasticity
G = modulus of rigidity, taken as 79300 N/mm^2 for carbon, low alloy and martensitic stainless steel
I = moment of inertia about bar cross section $\left(\frac{\pi d^4}{64} \text{ for round bars}\right)$
l = length of transition
l_r = equivalent length of transition
L = effective length of bar
L_c = length of cylindrical part of bar with diameter d
n = number of serrations
p = distance of point of load application from datum line (see Figure 2)
p_o = distance of point of load application from datum line when load (P) = 0 (see Figure 2)
P = load applied to lever
q = shear stress (solid torsion bar)
q_h = shear stress (hollow torsion bar)
r = transition radius
R = length of lever
$S = \frac{dP}{d\delta}$ = translational spring rate
$S_o = \frac{T}{\theta}$ = angular spring rate (solid torsion bar)
S_h = angular spring rate (hollow torsion bar)
T = torque
α = angle between datum line and lever arm in unloaded position (radians)
β = included angle of cone transition (radians)
ϕ = angle between lever arm and datum line (radians)
θ = applied angle of twist (angular deflection) of solid torsion bar (radians)
θ_h = applied angle of twist (angular deflection) of hollow torsion bar (radians)
δ = translational deflection (radians)

The torsion bar is an extremely efficient spring. As regards to the energy per unit of volume, it is surpassed only by springs, which are stressed in pure tension or compression, e.g. ring springs. Apart from proper design and great accuracy in manufacture, a principal condition for satisfactory service is that the bar be supported in such a way that it is stressed in pure torsion and be free from any bending.

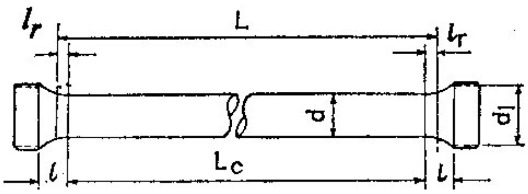

Figure 1

Design Calculations

(a) The Bar (see Figure 1)

If a cylindrical torsion bar of diameter d and effective length L, manufactured from a material with modulus of rigidity G is subjected to a torque T, the resulting twist is given by

$$\theta = \frac{32TL}{\pi d^4 G} \quad (1)$$

the corresponding surface shear stress by

$$q = \frac{16T}{\pi d^3} \quad (2)$$

and the angular spring rate by

$$S_o = \frac{T}{\theta} = \frac{\pi d^4 G}{32L} \quad (3)$$

(b) The Bar and Lever (see Figure 2)

For many purposes, e.g. for vehicle suspension, it is necessary to translate the twist of the bar into a linear displacement (translational deflection). This is usually done by attaching a lever to one end of the bar.

Although twist and stress are proportional to the torque and S_o is a constant, the torque, the translational deflection δ, and the stress do not depend on the load P alone but also on the position of the lever, relative to the datum line AA which is perpendicular to the direction of P and through the centre of the torsion bar. The translational spring rate S is not constant.

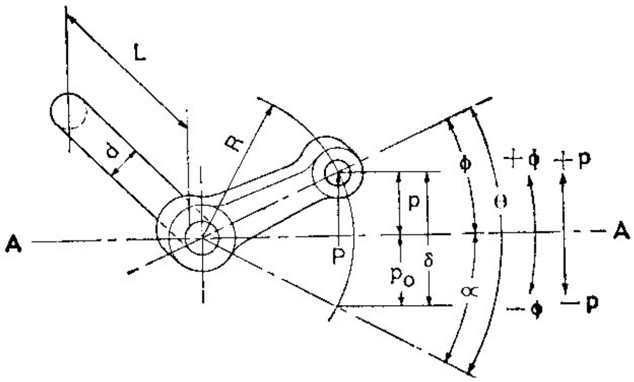

Figure 2

The position of the lever is given by p = R sin ϕ (4)

where p denotes the perpendicular distance from the effective end of the lever to the datum line AA and ϕ the angle between the lever axis and AA.

The combined bar-lever can be designed by means of the formulae:

$$P = \frac{T}{R\cos\phi} = \frac{S_o}{R\cos\phi}\,\theta = \frac{S_o}{R}\,\frac{(\alpha+\phi)}{\cos\phi} \quad (5)$$

$$S = \frac{dP}{d\delta} = \frac{dP}{dp} = \frac{S_o\left[1+(\alpha+\phi)\tan\phi\right]}{R^2\cos^2\phi} \quad (6)$$

From (5) and (6) $\alpha = \dfrac{1}{\dfrac{SR}{P}\cos\phi - \tan\phi} - \phi$ (7)

$$\delta = p_o + p = R\left(\sin\alpha + \sin\phi\right) \quad (8)$$

$$q = \frac{16}{\pi d^3} S_o\,\theta = 5.09\,\frac{S_o}{d^3}\left(\alpha+\phi\right) \quad (9)$$

Design of Bar Ends

As the ends of the bars have to be mounted, and as mounting involves additional stresses, the ends have to be made of larger diameter than the diameter of the rod proper (see Figure 1) either by machining from bar stock of larger size than the finished ends or (preferably) by upsetting.

(a) Serrated Ends

This is the most commonly used end connection for a torsion bar since it permits a smaller end diameter than any other system. Experience has shown that the root diameter of the serration's must always be greater than 1.15 d and preferably greater than 1.25 d when high working stress (q) is required, and that the length of the serration's should be equal to 0.4 d. The design of serration's is described in B.S. 2059 "Straight-sided splines and serration's". Formation of serration's may be carried out before or after heat treatment, depending on manufacturing procedure.

The hardness of the housing should be within the range 300-350 HV and the length of the serration's in it must be greater than those on the bar.

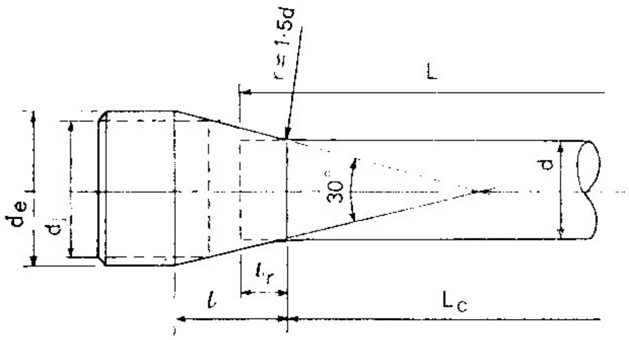

Figure 3 Typical Coned Transition Zone

(b) Hexagonal Ends

These ends can be manufactured without machining. The usual manufacturing procedure uses centreless ground bar which is cut to length, the ends are then upset into the hexagon shape and the sides of the hexagon finished with a coining operation to improve their flatness. For long life at high working stress (q) the distance across flats should be at least 1.4 d under less critical conditions a minimum value of 1.2 d may be used. A length of hexagon end of 0.7 d has been found to be satisfactory. The hardness of the housing should be within the range 270-300 HV.

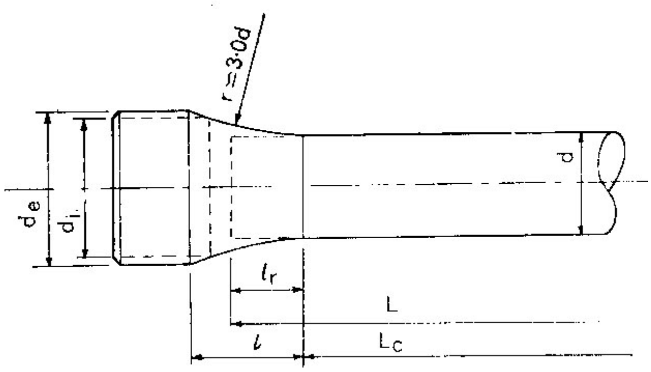

Figure 4 Typical Radiused Transition Zone

Transition between Bar and Shoulder

To avoid stress concentrations, the diameter must increase very gradually over the length (l) of transition from the cylindrical part of the bar to the shoulder. The increase may be obtained either by a slender cone (Figure 3 where $\beta = 30°$) or by a large radius (Figure 4) cones of 30° included angle and radii of 3.0 d being commonly used. The length (l) is dependent on the diameter (d) of the bar and the overall diameter of the serrated end (d_e).

(a) Coned transition with included angle β

$$l = \frac{1}{2} \cot \frac{\beta}{2} (d_e - d)$$

(b) Transition of radius r

$$l = (d_e - d)\sqrt{\frac{r}{d_e - d} - 0.25}$$

in the case of hexagonal ends, d_e should be taken as the width across the flats.

As the transition zone contributes to the twist to a lesser degree than a zone of equal length and of diameter d, it is convenient to reduce l to an equivalent length l_r of diameter d. Then the effective length of the bar is given by $L = L_c + 2l_r = L_c + 2al$ where for a coned transition, the reduction factor

$$a = \frac{1}{3}\left[\left(\frac{d}{d_e}\right) + \left(\frac{d}{d_e}\right)^2 + \left(\frac{d}{d_e}\right)^3\right]$$

and for a radius transition

$$a \approx \frac{1}{48}\left(\frac{d}{d_e}\right)^3\left[8 + 10\left(\frac{d_e}{d}\right) + 15\left(\frac{d_e}{d}\right)^2 + \frac{15\left(\frac{d_e}{d}\right)^3 \tan^{-1}\sqrt{\frac{d_e}{d} - 1}}{\sqrt{\frac{d_e}{d} - 1}}\right]$$

The reduction factor for both types of transition can be taken from the graph in Figure 5 where it is plotted against d_e/d. For hexagonal ends, d_e has to be replaced by the distance across the flats.

Hollow Torsion Bars

There are certain advantages in using a tubular bar instead of one, which is solid. The material near the axis of a solid torsion bar carries very little load and can therefore be discarded with a considerable saving in weight. Furthermore, when a space is limited the conventional solid bar can be replaced by a bar inside a tube, the two being clamped together at one end. The torque is then applied to the free end of the bar (or tube) whilst the remaining free end is clamped. In this arrangement the two loading points of the bar are brought close together and this can sometimes be a further advantage.

Formulae

For a hollow bar, equations 1, 2 and 3 become:

$$\theta_h = \frac{32TL}{\pi\left(d_o^4 - d_h^4\right)G} \; ; \; q_h = \frac{16Td_o}{\pi\left(d_o^4 - d_h^4\right)}$$

$$S_h = \frac{T}{\theta_h} = \frac{\pi\left(d_o^4 - d_h^4\right)G}{32L}$$

Equations 5, 6, 7 and 8 remain unchanged except that S_h and θ_h must replace S_o and θ but equation 9 becomes

$$q_h = \frac{16 S_h \theta_h d_o}{\pi\left(d_o^4 - d_h^4\right)}$$

where d_o = outside diameter of tube and
d_h = inside diameter of tube

Weight Saving

If W and W_h are the weight per unit length of solid and hollow bars respectively having the same outside diameter d_o, then

$$\frac{W_h}{W} = 1 - \left(\frac{d_h}{d_o}\right)^2$$

If the surface shear stresses are q and q_h, and θ and θ_h the angles of twist respectively, then for the same outside diameter d_o and torque T

$$\frac{q}{q_h} = \frac{\theta}{\theta_h} = 1 - \left(\frac{d_h}{d_o}\right)^4$$

In Figure 6 $\left(\frac{W_h}{W}\right)$ and $\left(\frac{q_h}{q}\right)$ are plotted against $\left(\frac{d_h}{d_o}\right)$ and it may be seen that the weight is reduced much more quickly than the stress increases so that for a weight reduction of 25% the stress is only increased by slightly over 5%.

Buckling

A very long bar subjected to pure torsion will become laterally unstable and buckle when the torque exceeds a certain critical value in the same way as a long strut under pure compression. This value depends on the modulus of elasticity, the moment of inertia and the bar length. The bar is stable if:

$$T \le \frac{2\pi EI}{L}$$

Manufacturing Specification

Prestressing

This process raises the permissible maximum operating stress by plastically deforming the bar in the direction of the load. It can only be applied to bars subject to loads, which are always in the same direction and the direction of loading should be marked on the end face of prestressed torsion bars. If the bars are used for vehicle suspensions where right-hand and left-hand side springs are stressed in opposite directions, it is advisable to make them non-interchangeable by appropriate design of end fittings. In the case of serrated ends, this can easily be done by making one matched pair with a tooth removed from the external serrations and omitting a space in the internal one.

Prestressing should result in a permanent set (angle of twist) of 0.01 L/d to 0.016 L/d radians which will be obtained by applying a twist of about 0.44 L/d radians and allowing springback to take place. The bar should be strained to the same position at least three times.

After prestressing, the bar may show a rate, which is somewhat lower than expected; it is due to a slight decrease in the shear modulus due to the cold work that has taken place. A value of G = 74470 N/mm^2 has been found to be applicable for carbon and low alloy steels after prestressing.

Shot Peening

For optimum fatigue properties shot peening of the bar (before or after prestressing) to a minimum Almen ‘A’ arc rise of 0.46mm is recommended. Conditioned cut wire shot or cast steel shot in the size range 0.76mm to 1.0mm is commonly used, but because of the necessity of peening the root radii of the serrations on the torsion bar it is important to ensure that a reasonable proportion of the shot has a diameter, which is less than half the root radii. No advantage is to be gained by shot peening bars, which are not subject to cyclic loading.

Material

The most commonly used material for highly stressed torsion bars in BS 970 Part 2, 251A58 silicon-manganese steel because of its superior hardenability over carbon steel. An alternative material is BS 970 Part 2, 735A51 chromium-vanadium steel. In either case the hardness of the finished bar should be 450-500 HV. Care must be taken in the manufacture and heat treatment to ensure freedom from decarburisation and other surface defects, which reduce the fatigue life of the bar. Inspection by magnetic crack detection is therefore usually specified.

Protection from corrosion is very important and should be provided immediately for torsion bars, which have been shot peened. Under a normal atmospheric environment, protection by special paints is generally considered adequate.

Design Example

The following parameters of a torsion bar are given:

effective length of lever	R	= 381mm
static load	P_1	= 11121N
translational spring rate at static load	S_1	= 87.6N/mm
position of lever at static load	p_1	= -127mm
dynamic ride deflection	$p_2 - p_1$	= 229mm
modulus of rigidity of material (after pre-stressing)	G	= 74470N/mm^2

(a) To find lever arm angle ϕ_2 at full bump. At static load position from (4)

$$\sin\phi_1 = \frac{p_1}{R} = \frac{-127}{381} = -0.333$$ static lever angle ϕ_1 = –0.340 radian.

From (7) the angular deflection from free position to the datum line is:

$$\alpha = \frac{1}{\frac{S_1 R}{P_1}\cos\phi_1 - \tan\phi_1} - \phi_1 = \frac{1}{\left(\frac{87.6 \text{ x } 381 \text{ x } 0.943}{11121}\right) - (-0.354)} - (-0.340) = 0.654 \text{ radian.}$$

the lever arm angle at full bump, from (4) is given by:

$$\phi_2 = \sin^{-1}\frac{p^2}{R} = \sin^{-1}\left(\frac{-127 + 229}{381}\right) = 0.270 \text{ radian.}$$

(b) To find the total angle of twist θ

$$\theta = \alpha + \phi_2 = 0.654 + 0.270 = 0.924 \text{ radian.}$$

(c) To find the maximum torque T_2 from (5)

$$S_o = \frac{P_1 R\cos\phi_1}{(\alpha + \phi_1)} = \frac{11121 \text{ x } 381 \text{ x } 0.943}{0.314} = 12724797 \text{Nmm / radian.}$$

Since S_o is constant, the maximum torque $T_2 = S_o\theta$ = 12724797 x 0.924 = 11757712Nmm.

(d) To find the bar diameter d from (2) $d = \sqrt[3]{\frac{16\,T_2}{\pi\,q_2}} = \sqrt[3]{\frac{16 \text{ x } 11757712}{\pi \text{ x } 830}} = 41.6\text{mm.}$

(e) To find the effective length of bar L from (3) $L = \frac{\pi\,d^4\,G}{32\,S_o} = \frac{\pi \text{ x } 41.6^4 \text{ x } 74470}{32 \text{ x } 12724797} = 1721\text{mm.}$

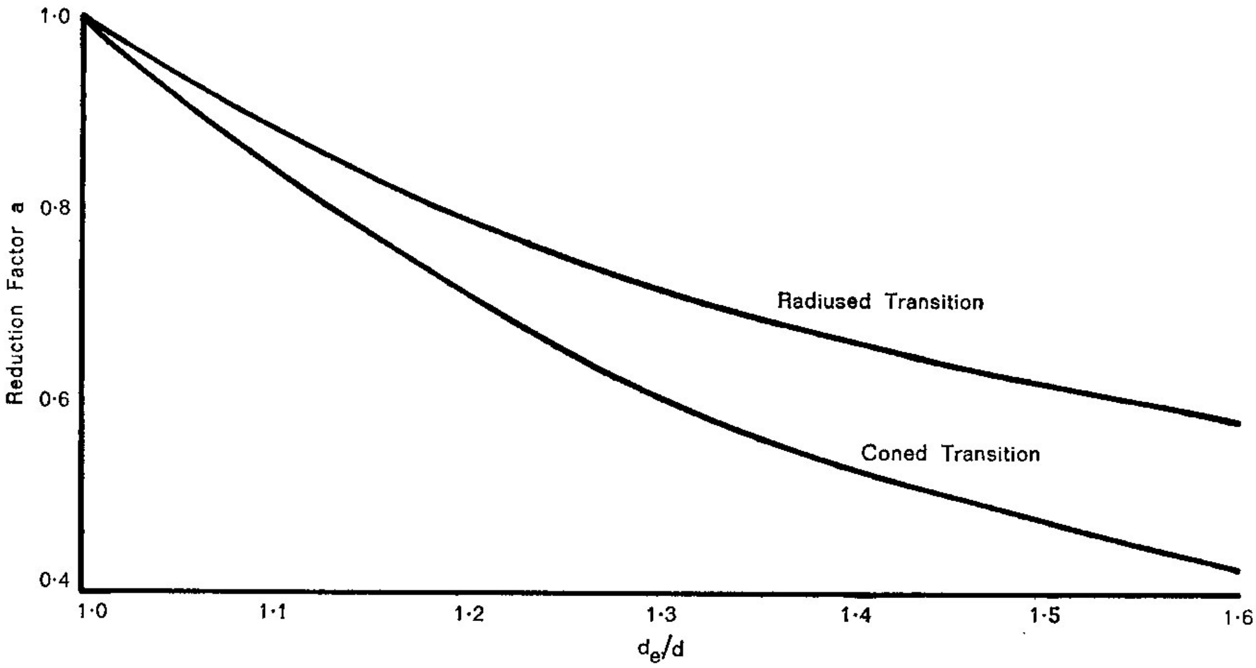

Figure 5 Reduction Factor for Serrated Ends

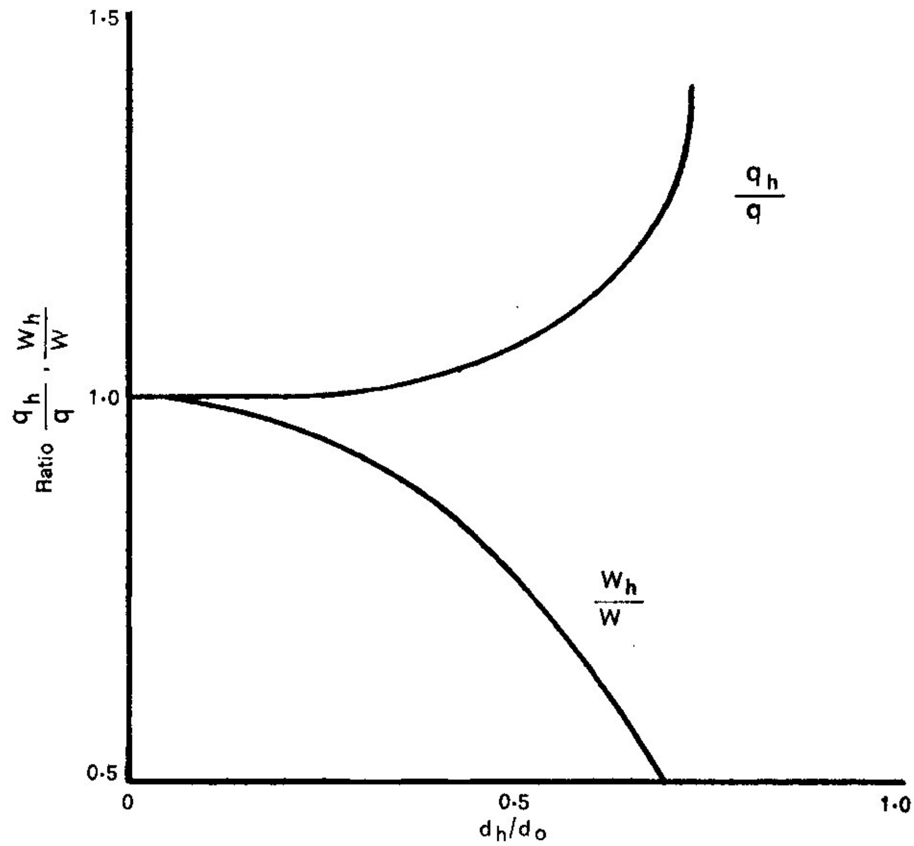

Figure 6